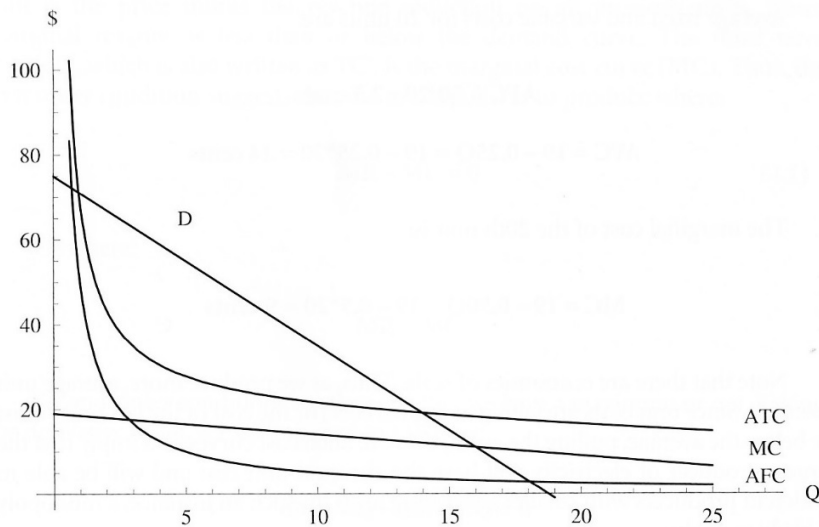


## Modeling Electricity Markets

Taken from *International Energy Markets*, Carol A. Dahl, 2004, pp. 91-97



Electricity markets are generally thought of as natural monopolies

- Declining ATC industries over the relevant range of production
- There exist economies of scale; as we produce more average unit costs fall
- Declining unit costs mean that marginal cost (the cost of the last unit) must be below the average, pulling the average down as Q increases
- The largest producer of electricity will have the lowest unit cost and thus be able to beat out similar producers
- Thus we're likely to get a natural monopoly

Monopolist can choose to produce at any point on the demand curve

Generally a monopolist will choose to maximize profits

Inverse demand curve:  $P = P(Q)$  which slopes downward:  $\partial P / \partial Q < 0$

i.e., the monopolist can sell at a higher price by reducing the quantity produced

Assume that the total cost  $[TC(Q)]$  curve slopes upward:  $\partial TC / \partial Q > 0$  which means that marginal costs are positive [since  $MC = \partial TC / \partial Q$ ]

But in this industry  $\partial^2 TC / \partial Q^2 < 0$ , i.e., the slope gets flatter as Q increases, which means that marginal costs decrease as Q increases

Monopolist's profits:  $\pi = P(Q) \cdot Q - TC(Q)$

Maximize profits by taking the derivative with respect to Q and setting it equal to zero

$$\frac{d\pi}{dQ} = P + \frac{dP}{dQ} \cdot Q - \frac{dTC}{dQ} = 0$$

Or  $MR - MC = 0$  i.e., profit maximization occurs at the level of output corresponding to  $MR = MC$

Consider the following example (could rescale to make the numbers more sensible; think of a micro power station of some sort in a small village):

Let P = price of electricity in cents per kWh

Q is the annual supply and demand of electricity measured in kWh

[Inverse] Demand curve:  $P = 75 - 4Q$

$$FC = 50$$

$$VC = 19Q - 0.25Q^2$$

$$TC = 50 + 19Q - 0.25Q^2$$

$$AFC = 50/Q$$

$$AVC = 19 - 0.25Q$$

$$MC = \frac{dTC}{dQ} = 19 - 2(0.25)Q^{2-1} = 19 - 0.50Q$$

Suppose  $Q = 20$  kWh (small market)

$$TC = 50 + 19 \times 20 - 0.25(20)^2 = 330 \text{ cents or } \$3.30$$

$$AFC = 50/20 = 2.5 \text{ cents}$$

$$AVC = 19 - 0.25(20) = 14 \text{ cents}$$

$$ATC = AFC + AVC = 16.5 \text{ cents @ } Q=20$$

$$\text{Marginal cost of the 20}^{\text{th}} \text{ unit: } MC = 19 - 0.5(20) = 9 \text{ cents}$$

Find  $Q_m$ , the profit maximizing level of output:

$$TR = (75 - 4Q) \cdot Q = 75Q - 4Q^2$$

$$MR = \frac{\partial TR}{\partial Q} = 75 - 8Q$$

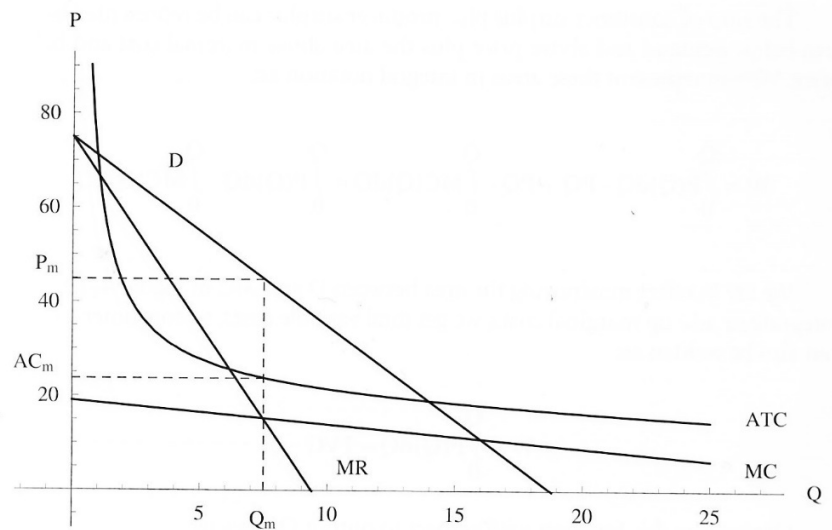
Setting  $MR = MC$  and solving for  $Q$  yields:

$$75 - 8Q = 19 - 0.5Q \text{ or } Q_m = 7.467$$

$$\text{and } P_m = 75 - 4(7.467) = 45.132$$

$$\text{and } \pi_m = 159.067$$

Since  $P_m$  (price people are willing to pay) exceeds the marginal cost of producing  $Q_m$  there are “social losses” associated with monopoly output.



We measure social welfare as the sum of consumer and producer surplus, i.e., the area below the demand curve and above the price plus the area above marginal cost and below price.

$$W = \int_0^{Q_m} P(Q) dQ - P_m \cdot Q_m + P_m \cdot Q_m - \int_0^{Q_m} MC(Q) dQ = \int_0^{Q_m} P(Q) dQ - \int_0^{Q_m} MC(Q) dQ$$

Maximizing social welfare ( $W$ ) essentially means maximizing the area between the demand curve and the marginal cost curve.

Adding up (integrating) marginal costs yields total variable costs so

$$W = \int_0^Q P(Q) dQ - TVC(Q)$$

And maximizing social welfare means

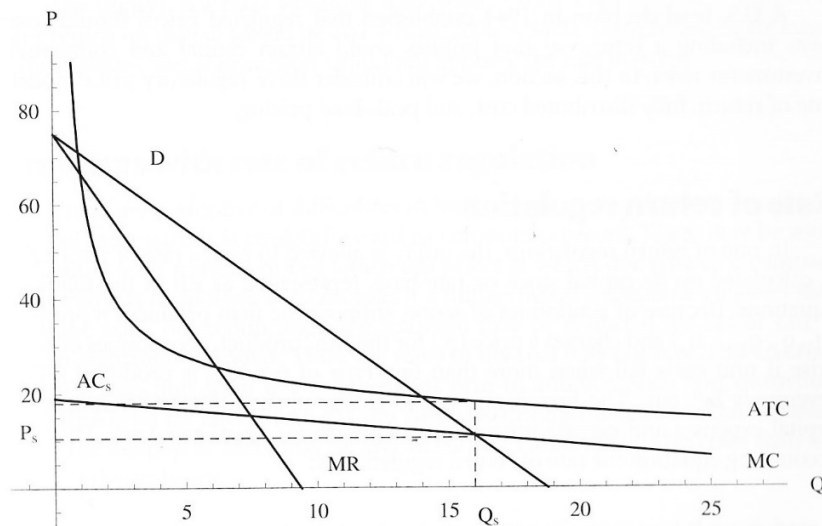
$$\frac{\partial W}{\partial Q} = \frac{\partial \left( \int_0^Q P(Q) dQ - TVC(Q) \right)}{\partial Q} = P(Q) - MC(Q) = 0$$

In other words, we maximize social welfare by choosing a level of output consistent with the point where price equals marginal cost

For the example above,  $P = MC = 75 - 4Q = 19 - 0.5 \cdot Q$  or  $Q_s = 16$  and  $P_s = 5 - 4(16) = 11$

Since  $ATC > P$  @  $Q_s$  the electric utility loses money:

$$\pi_s = PQ - TC = 11 \cdot 16 - 50 - 19(16) + 0.25(16)^2 = -\$114$$



Role of government is to regulate prices so that the utility stays in business

Some options:

- Rate of return regulation – PUC sets prices so that the utility can earn a “normal” rate of return on its capital investments plus cover operating costs (1944 court case). Normal rate of return should be enough to attract investors by compensating them for the inherent risks of operating a utility.
- Peak-load pricing – cover the cost of idle capital
- Utility cost allocation – distribute fixed costs across consumer groups in a way that reflects their usage.